# A Modified Inertial Iterative Algorithms for Solving Split Common Fixed Point Problems in real Hilbert Spaces

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D.O.I: 10.56201/ijasmt.v8.no3.2022.pg1.15

***Abstract***
*We introduce and study an inertial-based iterative algorithm for solving split common fixed point problem involving a certain class of nonlinear mapping in real Hilbert spaces. Under some mild assumptions, we obtain a strong convergence result of the proposed algorithm. Our result improves and extends newly announced results in this area.*

***2010 Subject Classification: 47H05; 47H09; 49J53; 90C25***

***Key words**. Metric projection; nonexpansive mappings; parallel algorithm; split common fixed point problem.*

## 1. Introduction

The theory of nonlinear functional analysis is quite rich and a fascinating area of research, in particular, the fixed point theory. The theory of fixed point can be classified into theories for the existence of fixed points and approximation of fixed points. It has proven to be at the heart of technological development and numerous applications ranging from engineering, computer science, mathematical sciences and social sciences. This is due to the fact that, many real world problems after transforming them into mathematical equations may not have analytic solutions. Hence, seeking for existence and uniqueness of solutions, fixed point of a certain class of operators and approximation of solutions of such problems has flourishing areas of research for many decades. The Banach Contraction Mapping Principle is one of the cornerstones in this high level of achievement. In this recent time, fixed point theory has successfully been employed in Data science, Machine Learning and Artificial intelligence to mention but a few (for this updates, the reader is encouraged to consult [31, 32,33, 34]. One of the powerful tools that paved ways for this modern development is the use of nonexpansive operators. A self-map $T$ define on $C$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C,$ where $C$ is a nonempty closed subset of a real Hilbert space, $H$. However, a point $x^* \in C$ is said to be the fixed point of $T$ if $Tx^* = x^*$.

One of the most fruitful areas where fixed points of nonexpansive mappings have been successfully employed is in signal processing and image recovery which are called inverse problems. Among many research published articles in this direction, Censor and Elfving [1] published their most influential paper where they introduced and studied split feasibility problem (SFP) for modeling inverse problems arising in phase retrieval and medical imaging. The problem is of the form:

$$\begin{cases} \text{find } x^* \in C \text{ such that } y^* = Ax^* \in Q, \\ \quad\quad \text{equivalently}, \\ \text{find } x^* \in S = C \cap A^{-1} Q, \end{cases} \quad (1.1)$$

where $C$ and $Q$ are two nonempty closed and convex subsets of Hilbert spaces $H_1$ and $H_2$ respectively, $A: H_1 \rightarrow H_2$ a bounded linear operator and $A^*: H_2 \rightarrow H_1$, its adjoint. Split Feasibility Problem (1) was firstly introduced and studied by Censor and Elfving [1] in 1994 for the purpose of modeling some inverse problems. Its wide applications in signal processing and image reconstructions (see for details [2,3]) drawn the attention of scientists and subsequently opened up for researches for over two decades now, for instance, see Byrne [2,3], Censor et al. [1, 4-6], Xu [7-8], Masad and Reich [9], Shehu et al. [10].

In 1992, Eicke [11], introduced a special type of SFP called convexly constrained linear inverse problem. It can be formulated as follows:
given $b \in Q$,

$$\begin{cases} \text{find } x^* \in H_1 \text{ such that } Ax^* = b, \\ \quad\quad equivalently, \\ \text{find } x^* \in H_1 \text{ such that } x^* = A^{-1}b \end{cases} \quad (1.2)$$

The proposed algorithm provided by [1] for solving (1.1) was faced with a challenge of matrix inversion. Due to difficulty to compute the matrix inversion, the most popular iterative scheme for SFP is Byrne's CQ methods [2] which did not involve matrix inversion but projections onto closed convex sets. Thus, it is defined as follows: for any starting point $x_0 \in H_1$ and for all $n \geq 0$, the recursive sequence $\{x_n\}$ is given by

$$x_{n+1} = P_C^{H_1}(x_n - \gamma A^*(I^{H_2} - P_Q^{H_2})Ax_n), \quad (1.3)$$

where $P_C^{H_1}$ and $P_Q^{H_2}$ are the metric projections onto C and Q respectively and $\gamma \in \left(0, \frac{2}{\|A\|^2}\right)$.

Following (1.1) and (1.2) the Split Feasibility Problem (SFP) (see [35,376] for details) is formulated as follows:

$$\begin{cases} find\ x^* \in H_1\ that\ solves\ IP_1 \\ \quad\quad such\ that \\ y^* = Ax^* \in H_2\ solves\ IP_2, \end{cases} \quad (1.4)$$

where $IP_1$ and $IP_2$ define two inverse problems in $H_1$ and $H_2$ respectively.

Being at the hub of modern research, problem (1.1) has been modified and generalized in different capacities by many researchers. For instance, Censor et al. [4] generalized (1.1) to multiple set split feasibility problem (MSSFP) which requires finding a point closest to a family of closed convex sets in one space such that its image under a linear transformation will be closest to another family of closed convex sets in the image space. That is, given two closed and convex subsets $C_i, i = 1,2, \dots, N$ and $Q_j, j = 1,2, \dots, M$ of $H_1$ $and$ $H_2$ respectively. Let $T_i: H_1 \rightarrow H_1, i = 1,2, \dots, N$ and $S_j: H_2 \rightarrow H_2, j = 1,2, \dots, M$ be two finite families of mappings and $A: H_1 \rightarrow H_2$ be a bounded linear operator. Then the MSSFP is formulated as follows:

$$\begin{cases} \quad find\ x^* \in H_1\ such\ that \\ x^* \in (\cap_{i=1}^N C_i) \cap A^{-1}(\cap_{j=1}^M Q_j). \end{cases} \tag{1.5}$$

In 2009, Censor and Segal [5] generalized (1.5) with the concept of Split Common Fixed Point Problem (SCFPP) which is formulated as follows:

$$\text{find } x^* \in \Omega := (\cap_{i=1}^N F(T_i) \cap A^{-1}(\cap_{j=1}^M F(S_j)) \neq \emptyset. \tag{1.6}$$

One of the striking applications of SCFPP is on the intensity-modulated radiation therapy (IMRT), which have attracted attention of many researchers in this area. As a result of that, in recent years SCFPP has been extensively studied and extended by many authors (see for example, [12-18]). More extensions of this research can also be found in [19-21] and references contained therein.

In 2019, Reich et al. [22] introduced and studied parallel iterative methods for solving the SCFPP in real Hilbert spaces and obtained strong convergence in the proposed algorithms. In recent years, there has been a tremendous interest in developing the fast convergence of algorithms, especially for the inertial type extrapolation method, which was first proposed by Polyak in [37]. This inertial technique is based upon a discrete analogue of a second order dissipative method. This method was not known until the Nesterov's acceleration gradient methods was published in 1983 (see, [38]) and by 2009, Beck and Teboulle [39] made it very popular. Recently, some researchers have constructed different fast iterative algorithms by means of inertial extrapolation techniques, for example, [40-41].

Motivated and inspired by the recent of work of Reich et al. (2019), we proposed a new algorithm using the Halpern iterative method and obtained strong convergence result. Our major contributions include:

- A new inertial method for solving the SCFPP in a real Hilbert space. In the spirit of Polyak, our proposed method is more efficient when compared with [22].
- We incorporated a more general class of mappings called demi-contractive mapping. Many important operators like nonexpansive mapping, pesudocontractivs, $k$-strictly pseudocontractive, quasi-nonexpansive operators among many others are all embedded in demicontractive mapping (see, remark 2.1 below).

The rest of the paper is organized as follows: section two deals with basic definitions and Lemmas very relevant to our work. In section three, we state and establish the proof of our proposed method and as well an important corollary as an immediate consequences of algorithm.

## 2. Preliminaries

Let H be a real Hilbert space and C, a nonempty, closed and convex subset of H in which $\langle , \rangle$ denotes the inner product and for any vector $x \in H$, $\|x\| = \sqrt{\langle x, x \rangle}$, is the induced norm. More so, for any, $x \in H$ there exists a unique nearest point $P_C^H(x) \in C$ such that

$$\|x - P_C^H(x)\| = \inf_{q \in C} \|x - q\|. \tag{2.1}$$

The mapping $P_C^H$ defined on (2.1) is called the metric projection from H onto C. The following properties characterized $P_C^H$:

$$P_C^H \in C \text{ and } \langle P_C^H x - x, P_C^H x - y \rangle \leq 0, \forall y \in C. \tag{2.2}$$

It follows from (2.2) that

$$\|x - P_C^H\|^2 + \|y - P_C^H\|^2 \leq \|x - y\|^2, \forall x \in H, \forall y \in C. \tag{2.3}$$

It is well known that a mapping $U: H \to H$ is called:

1). nonexpansive if $\|Ux - Uy\| \leq \|x - y\|, \forall x, y \in H$ 2). $\gamma$ –strictly pseudocontractive with $0 \leq \gamma < 1$ if

$\|Ux - Uy\|^2 \leq \|x - y\|^2 + \gamma\|(I - U)x - (I - U)y\|^2, \forall x, y \in H.$ (2.4)

3) $\beta$ −demicontractive with $0 \leq \gamma < 1$ if

$\|Ux - z\|^2 \leq \|x - z\|^2 + \beta\|(I - U)x\|^2$ for all $x \in H$ and for all $z \in F(U)$. (2.5)

or equivalently

$\langle Ux - x, x - z\rangle \leq \frac{\beta-1}{2}\|x - Ux\|^2, for\ all\ x \in H, for\ all\ z \in F(U).$ (2.6)

or equivalently

$\langle Ux - z, x - z\rangle \leq \frac{\beta-1}{2}\|x - Ux\|^2, for\ all\ x \in H, for\ all\ z \in F(U).$

**Remark 2.1:** We quickly observe that the class of nonexpansive mapping and strictly pseudocontractive are embedded in the class of demicontractive mapping.

**Example 2.1 [31]: a.** Let $H$ be the real line and $C = [-1, 1]$. Define $U$ on $C$ be

$$\begin{cases} \frac{2}{3}x \sin\frac{1}{x}, if\ x \neq 0; \\ 0, \quad if\ x = 0. \end{cases}$$

The following example shows that there exists a demicontractive operator but it is not strictly pseudocontractive.

b. $T: [-2,1] \to [-2,1], Tx = -x^2 - x.$

The following example shows that there exists a demicontractive operator but it is not quasi-nonexpansive and strictly pseudocontractive.

**c.** Let $\mathcal{H}$ denote the real line with usual norm and $T: \mathcal{H} \to \mathcal{H}$ be a function defined by

$$Tx = \begin{cases} x, & if -\infty < x < 0 \\ -3x, & if\ 0 \leq x < +\infty. \end{cases}$$

Recall that an operator $A: H \to 2^H$ is called a monotone if for all $x, y \in D(A)$, we get $\langle u - v, x - y\rangle \geq 0$ for all $u \in A(x)$ and $v \in A(y)$. We define the domain of A, the image and the graph of A as follows:

$$D(A) := \{x \in H: A(x) = \emptyset\},$$
$$R(A) := \cup \{A(z): z \in D(A)\} \text{ and}$$
$$G(A) := \{(x,y) \in H \times H: x \in D(A), y \in A(x)\}.$$

and for any $x \in A^{-1}(y) \Leftrightarrow y \in A(x)$

Recall also that an operator $A$ is said to be a maximum monotone if it is not contained in any other monotone. Equivalently, following the theorem of Minty [25], $A$ is maximum operator if and only if $R(I + \lambda A) = H$, where I is an identity operator on $H$ and $\lambda > 0$. With respect to $\lambda\ and\ I$, a single-valued and a nonexpansive map

$$J_\lambda^A: R(I + \lambda A) \to D(A)$$

defined by $J_\lambda^A := (I + \lambda A)^{-1}$ is a called the resolvent of $A$.

The following lemmas are well known and are needed for our result.

**Lemma 2.1:** Let H be a real Hilbert space. Then for all $x, y \in H$ and for any $t \in \mathcal{R}$, we have

i) $\|x - y\|^2 = \|x\|^2 + \|y\|^2 - 2\langle x, y\rangle,$

ii) $\|x+y\|^2 = \|x\|^2 + \|y\|^2 + 2\langle x,y\rangle$,
iii) $\|tx+(1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x-y\|^2$.

**Lemma 2.2 [27]:** Assume $T: H \to H$ is a nonexpansive mapping. Then $(I-T)$ is demiclosed on $H$ if whenever $\{x_n\} \subset H$ converges weakly to some $x \in H$ and the sequence $\{(I-T)x_n\}$ converge strongly to y, we get that $(I-T)x = y$.

**Lemma 2.3 (Opial):** Let C be a nonempty set of H and$\{x_n\}$ be a sequence in H such that:
a) for every $x \in C$, $\lim_{n\to\infty}\|x_n - x\|$ exists;
b) every sequential weak cluster point of $\{x_n\}$ is in C;
Then $\{x_n\}$ converges weakly to a point in C.

**Lemma 2.4 [28]:** Consider a nondecreasing at infinity sequence of real numbers $\{s_n\}$. That is, there exists a subsequence $\{s_{n_k}\}$ such that

$$s_{n_k} \leq s_{n_k+1}, \forall k \geq 0.$$

Define an integer sequence $\{\tau(n)\}$ for all $n > n_0$, by

$$\tau(n) \coloneqq \max\{n_0 \leq k \leq n : s_k < s_{k+1}\}.$$

Then $\tau(n) \to \infty\ as\ n \to \infty$ and for all $n > n_0$, we obtain

$$\max\{s_{\tau(n)}, s_n\} \leq s_{\tau(n)+1}.$$

**Lemma 2.5 [29]:** Let $\{s_n\} \subset [0,+\infty), \{\alpha_n\} \subset (0,1)\ and\ \{c_n\}$, a sequence of real numbers satisfying the following two conditions:
i) $s_{n+1} \leq (1-\alpha_n)s_n + \alpha_n c_n$;
ii) $\sum_{n=0}^{\infty}\alpha_n = \infty, \limsup_{n\to\infty} c_n \leq 0$.
Then, $\lim_{n\to\infty} s_n = 0$.

## 3. The Main Results

**Theorem 3.1:** Let $H_1$ and $H_2$ be two real Hilbert spaces. Let $T_i: H_1 \to H_2, 1 = 1,2,\ldots,N$, and $S_j: H_2 \to H_2, j = 1,2,\ldots,M$, be demicontractives mappings respectively. Let $A: H_1 \to H_2$ be a bounded linear operator. Assume that $\Omega \coloneqq (\bigcap_{i=1}^{N} F(T_i)) \bigcap A^{-1}(\bigcap_{j=1}^{M} F(S_j)) \neq \emptyset$. We consider the problem of the following:

$$\text{find } x^* \in \Omega. \tag{3.1}$$

For any $u, x_0 \in H_1$, We define $\{x_n\}$, the sequence generated by the following algorithms by

$$\begin{cases} w_n = x_n + \theta_n(x_n - x_{n-1}), \\ y_n = \sum_{i=1}^{N} a_{i,n}\tilde{T}_i w_n, \\ z_n = \sum_{j=1}^{M} b_{j,n} S_j(Ay_n), \\ t_n = y_n + \delta A^*(z_n - Ay_n), \\ x_{n+1} = \alpha_n u + (1-\alpha_n)t_n, n \geq 0, \end{cases} \tag{3.2}$$

where $\{a_{i,n}\}, i = 1,2,\ldots,N, \{b_{j,n}\}, j = 1,2,\ldots,M$ and $\{\theta_n\}$ are sequences of positive real numbers while $\tilde{T}_i \coloneqq \beta_{i,n}I + (1-\beta_{i,n})T_i$ and $\{\beta_{i,n}\} \subset (0,1), for\ i = 1,2,\ldots,N.\ 0 \leq \theta \leq \theta_n \leq \overline{\theta_n}$
where $\overline{\theta_n}$ is defined by

$$\overline{\theta_n} := \begin{cases} \min\left\{\dfrac{n-1}{n+\theta-1}, \dfrac{\mu_n}{\|x_n - x_{n-1}\|}\right\} & if\ x_n \neq x_{n-1} \\ \dfrac{n-1}{n+\theta-1} & otherwise. \end{cases}$$

Then the sequence $\{x_n\}$ generated by theorem (3.2) converges strongly to point $x^* = P_{\Omega}^{H_1}(u)\ as\ n \to \infty$ if the following five conditions are satisfied:

C1) $\{a_{i,n}\}, \{b_{i,n}\} \subset [a,b] \subset (0,1)$ for all $i = 1,2,\dots,N, j = 1,2,\dots,M$ and $\sum_{i=1}^{N} a_{i,n} = \sum_{i=1}^{N} b_{i,n} = 1$ for all $n \geq 1$;

C2) $\{\beta_{i,n}\} \subset [c,d] \subset (0,1)$ for all $i = 1,2,..,N$;

C3) $\lim_{n\to\infty} \alpha_n = 0$ and $\sum_{n=1}^{\infty} \alpha_n = \infty$;

C4 $\delta \in (0, \frac{1}{\|A\|^2})$

C5 $\lim_{n\to\infty} \frac{\mu_n}{\beta_n} = 0\ and\ \lim_{n\to\infty} \beta_n = 0.$

**Proof.** We divide the proof into six parts

**Step 1.** We prove that $\{x_n\}$ is bounded.

For any $p \in \Omega$, it follows that $T_i(p) = p, \forall i = 1,2,\dots,N\ and\ Ap = F(S_j), j = 1,2,\dots,M$ we obtain from convexity of $\|.\|^2$ that

$\theta_n\|x_n - x_{n-1}\| \leq \mu_n$ for all $n \in \mathbb{N}$, which implies from (C5) that $\frac{\theta_n}{\beta_n}\|x_n - x_{n-1}\| \leq \frac{\mu_n}{\beta_n} \to 0$, as $n \to \infty$.

Hence, there exist $M_1 > 0$ such that

$$\frac{\theta_n}{\beta_n}\|x_n - x_{n-1}\| \leq M_1, \forall n \in \mathbb{N}.$$

Thus, using the last inequality, we get

$$\|w_n - p\| \leq \|x_n - p\| + \theta_n\|x_n - x_{n-1}\| \quad (3.3)$$

$$= \|x_n - p\| + \beta_n \frac{\theta_n}{\beta_n}\|x_n - x_{n-1}\|$$
$$\leq \|x_n - p\| + \beta_n M_1.$$

Also,

$$\|y_n - p\|^2 = \left\|\sum_{i=1}^{N} a_{i,n}\widetilde{T}_i w_n - p\right\|^2$$

$$\leq \sum_{i=1}^{N} a_{i,n}\left\|\widetilde{T}_i w_n - p\right\|^2$$

$$= \sum_{i=1}^{N} a_{i,n}\left\|(\beta_{i,n}I + (1-\beta_{i,n})T_i)w_n - p\right\|^2 \quad (3.4)$$

$$= \sum_{i=1}^{N} a_{i,n}\left\|\beta_{i,n}(w_n - p) + (1-\beta_{i,n})(T_i w_n - p)\right\|^2$$

$$= \sum_{i=1}^{N} a_{i,n}\,[\beta_{i,n}\|w_n - p\|^2 + (1-\beta_{i,n})\|T_i w_n - p\|^2$$
$$-\beta_{i,n}(1- \beta_{i,n})\|w_n - T_i w_n\|^2]$$

$$\leq \sum_{i=1}^{N} a_{i,n} \, [\beta_{i,n} \|w_n - p\|^2 + (1-\beta_{i,n})[\|w_n - p\|^2 + \gamma \|w_n - T_i w_n\|^2]$$
$$-\beta_{i,n}(1-\beta_{i,n})\|w_n - T_i w_n\|^2]$$
$$= \sum_{i=1}^{N} \|w_n - p\|^2 - (\beta_{i,n}(1-\beta_{i,n}) - \gamma) \, \|(I - T_i)w_n\|^2$$
$$= \|w_n - p\|^2 - (\beta_{i,n}(1-\beta_{i,n}) - \gamma) \sum_{i=1}^{N} a_{i,n} \, \|(I - T_i)w_n\|^2.$$

It follows from (3.4) that

$$\|y_n - p\|^2 \leq \|w_n - p\|^2. \tag{3.5}$$

From the Algorithm and from the convexity of $\|.\|^2$, we get

$$\|t_n - p\|^2 \leq \|y_n - p\|^2 + \delta^2 \|A\|^2 \|z_n - Ay_n\|^2$$
$$+\delta\langle Ay_n - Ap, z_n - Ay_n\rangle \tag{3.6}$$

Since, $z_n$ and $Ay_n$ are in $H_2$, we get that

$$\|z_n - p\|^2 = \left\|\sum_{j=1}^{M} b_{j,n} \, (S_j(Ay_n) - Ay_n)\right\|^2$$
$$\leq \sum_{j=1}^{M} b_{j,n} \left\|S_j(Ay_n) - Ay_n\right\|^2. \tag{3.7}$$

Using Lemma 2.2 (i), the nonexpansivity of $S_j$ and the equalities $S_j(Ap) = Ap$, we estimate that

$$\langle Ay_n - Ap, z_n - Ay_n\rangle = \sum_{j=1}^{M} b_{j,n} \, \langle Ay_n - Ap, S_j(Ay_n) - Ay_n\rangle$$
$$= \tfrac{1}{2}\sum_{j=1}^{M} b_{j,n} \left(\left\|S_j(Ay_n) - Ap\right\|^2 - \|Ay_n - Ap\|^2 - \left\|S_j(Ay_n) - Ay_n\right\|^2\right)$$
$$= \frac{1}{2}\sum_{j=1}^{M} b_{j,n} \left(\left\|S_j(Ay_n) - S_j(Ap)\right\|^2 - \|Ay_n - Ap\|^2 - \left\|S_j(Ay_n) - Ay_n\right\|^2\right)$$
$$\leq \frac{1}{2}\sum_{j=1}^{M} b_{j,n} \left(\|Ay_n - Ap\|^2 - \|Ay_n - Ap\|^2 - \left\|S_j(Ay_n) - Ay_n\right\|^2\right)$$
$$-\tfrac{1}{2}\sum_{j=1}^{M} b_{j,n} \left\|S_j(Ay_n) - Ay_n\right\|^2 \tag{3.8}$$

Using (3.5)-(3.8), we estimate that

$$\|t_n - p\|^2 \leq \|y_n - p\|^2 - \delta(1 - \delta\|A\|^2) \sum_{j=1}^{M} b_{j.n} \left\|S_j(Ay_n) - Ay_n\right\|^2$$
$$\leq \|y_n - p\|^2. \tag{3.9}$$

It follows from (3.5) that

$$\|y_n - p\|^2 \leq \|w_n - p\|^2 \Longrightarrow \|y_n - p\| \leq \|w_n - p\|. \tag{3.10}$$

Hence, using (3.9)-(3.10), we get

$$\|t_n - p\|^2 \leq \|w_n - p\|^2. \quad (3.11)$$

Using the Algorithm, (3.3) and (3.11), we get

$$\begin{aligned}
\|x_{n+1} - p\| &= \|\alpha_n u + (1-\alpha_n)t_n - p\| \\
&\leq \alpha_n\|u - p\| + (1-\alpha_n)\|t_n - p\| \\
&\leq \alpha_n\|u - p\| + (1-\alpha_n)\|w_n - p\| \\
&\leq \alpha_n\|u - p\| + (1-\alpha_n)\|x_n - p\| + \theta_n\|x_n - x_{n-1}\| \quad (3.12) \\
&\leq \max\{\|x_n - p\|, \|u - p\|\} + M \\
&\vdots \\
&\leq \max\{\|x_1 - p\|, \|u - p\|\} + M,
\end{aligned}$$

where $M := \sum_{n=1}^{\infty} \theta_n \|x_n - x_{n-1}\| < \infty$.
Therefore, the sequence $\{x_n\}$ is bounded. Consequently, $\{w_n\}, \{y_n\}, \{z_n\}$ and $\{t_n\}$ are all bounded sequences.

**Step 2:** $\lim_{n\to\infty} x_n = x^* = P_{\Omega}^{H_1} u$.
Let $x^* = P_{\Omega}^{H_1} u$. Now, using the estimates in (3.4) and (3.9), we get

$$\begin{aligned}
\|x_{n+1} - x^*\|^2 &= \|\alpha_n(u - x^*) + (1-\alpha_n)(t_n - x^*)\|^2 \\
&\leq \alpha_n\|u - x^*\|^2 + (1-\alpha_n)\|t_n - p\|^2 \\
&\leq \alpha_n\|u - x^*\|^2 + \|t_n - p\|^2 \\
&\leq \alpha_n\|u - x^*\|^2 + \|w_n - p\|^2 - \big(\beta_{i,n}(1-\beta_{i,n}) - \gamma\big)\sum_{i=1}^{N} a_{i,n}\,\|(I - T_i)w_n\|^2 \\
&\quad -\delta(1 - \delta\|A\|^2)\textstyle\sum_{j=1}^{M} b_{j.n}\big\|S_j(Ay_n) - Ay_n\big\|^2. \quad (3.13)
\end{aligned}$$

See that

$$\begin{aligned}
\|w_n - p\|^2 &= \|x_n - p + \theta_n(x_n - x_{n-1})\|^2 \\
&= \|x_n - p\|^2 + \theta_n^2\|x_n - x_{n-1}\|^2 + 2\theta_n\langle x_n - p, x_n - x_{n-1}\rangle \\
&= \|x_n - p\|^2 + \theta_n^2\|x_n - x_{n-1}\|^2 - 2\theta_n\langle p - x_n, x_n - x_{n-1}\rangle \\
&\leq \|x_n - p\|^2 + \theta_n^2\|x_n - x_{n-1}\|^2 \\
&= \|x_n - p\|^2 + [\theta_n\|x_n - x_{n-1}\|]^2 \\
&= \|x_n - p\|^2 + [\beta_n\frac{\theta_n}{\beta_n}\|x_n - x_{n-1}\|]^2 \\
&\leq \|x_n - p\|^2 + [\beta_n M_1]^2 \quad (3.14)
\end{aligned}$$

Using (3.13) and (3.14), we get

$$\begin{aligned}
\|x_{n+1} - x^*\|^2 &\leq \|u - p\|^2 + \|x_n - p\|^2 + [\beta_n M]^2 - \big(\beta_{i,n}(1-\beta_{i,n}) - \gamma\big)\sum_{i=1}^{N} a_{i,n}\,\|(I - T_i)w_n\|^2 \\
&\quad -\delta(1 - \delta\|A\|^2)\textstyle\sum_{j=1}^{M} b_{j.n}\big\|S_j(Ay_n) - Ay_n\big\|^2 \\
&= \|u - p\|^2 + \|x_n - p\|^2 + [\beta_n M]^2 - [\big(\beta_{i,n}(1-\beta_{i,n}) - \gamma\big)\textstyle\sum_{i=1}^{N} a_{i,n}\,\|(I -
\end{aligned}$$

$$T_i)w_n\|^2 + \delta(1-\delta\|A\|^2)\sum_{j=1}^{M} b_{j.n}\left\|S_j(Ay_n) - Ay_n\right\|^2] \qquad (3.15)$$

It follows from (3.15) that

$$\left(\beta_{i,n}(1-\beta_{i,n}) - \gamma\right)\sum_{i=1}^{N} a_{i,n}\,\|(I-T_i)w_n\|^2 + \delta(1-\delta\|A\|^2)\sum_{j=1}^{M} b_{j.n}\left\|S_j(Ay_n) - Ay_n\right\|^2$$
$$\leq \|u-p\|^2 + \|x_n - p\|^2 - \|x_{n+1} - x^*\|^2 + [\beta_n M]^2 \qquad (3.16)$$

Observe that

$$\begin{aligned}\|x_{n+1} - x^*\|^2 &= \langle \alpha_n u + (1-\alpha_n)t_n - x^*, x_{n+1} - x^*\rangle \\ &= (1-\alpha_n)\langle t_n - x^*, x_{n+1} - x^*\rangle + \alpha_n\langle u - x^*, x_{n+1} - x^*\rangle \\ &\leq (1-\alpha_n)\|t_n - x^*\|.\|x_{n+1} - x^*\| + \alpha_n\langle u - x^*, x_{n+1} - x^*\rangle \\ &\leq \frac{(1-\alpha_n)}{2}(\|t_n - x^*\|^2 + \|x_{n+1} - x^*\|^2) + \alpha_n\langle u - x^*, x_{n+1} - x^*\rangle \qquad (3.17)\end{aligned}$$

Thus, (3.17) implies that

$$2\|x_{n+1} - x^*\|^2 \leq (1-\alpha_n)\|t_n - x^*\|^2 + (1-\alpha_n)\|x_{n+1} - x^*\|^2 + 2\alpha_n\langle u - x^*, x_{n+1} - x^*\rangle.$$

So, using (3.11) and (3.14), we obtain

$$\begin{aligned}\|x_{n+1} - x^*\|^2 &\leq (1-\alpha_n)\|t_n - x^*\|^2 - \alpha_n\|x_{n+1} - x^*\|^2 + 2\alpha_n\langle u - x^*, x_{n+1} - x^*\rangle, \\ &\leq (1-\alpha_n)\|t_n - x^*\|^2 + 2\alpha_n\langle u - x^*, x_{n+1} - x^*\rangle \qquad (3.18) \\ &\leq (1-\alpha_n)\|w_n - x^*\|^2 + 2\alpha_n\langle u - x^*, x_{n+1} - x^*\rangle \\ &\leq (1-\alpha_n)\|x_n - x^*\|^2 + [\beta_n M]^2 + 2\alpha_n\langle u - x^*, x_{n+1} - x^*\rangle \\ &= (1-\alpha_n)\|x_n - x^*\|^2 + \alpha_n[\frac{[\beta_n M]^2}{\alpha_N} + 2\langle u - x^*, x_{n+1} - x^*\rangle].\end{aligned}$$

Denote $\varphi_n := \|x_n - x^*\|^2$ and $\vartheta_n \coloneqq [\frac{[\beta_n M]^2}{\alpha_n} + 2\langle u - x^*, x_{n+1} - x^*\rangle]$. Then (3.17) can be rewritten in the form

$$\varphi_{n+1} \leq (1-\alpha_n)\varphi_n + \alpha_n\vartheta_n. \qquad (3.19)$$

It can be seen that to show $x_n \to 0$, we simply prove that $\varphi_n \to 0$ which requires the following two cases.

**Case 1:** It is seen that the sequence $\{\varphi_n\}$ is nonincreasing. Thus, there exists a natural number $N_0$ such that $\{\varphi_n\}$ is nonincreasing for $n \geq N_0$ such that $\{\varphi_n\}$ is a convergent sequence.
Since $\{\varphi_n\}$ is convergent, that is $\{\|x_n - x^*\|^2\}$, clearly

$$\lim_{n\to\infty}(\|x_n - x^*\|^2 - \|x_{n+1} - x^*\|^2) = 0. \qquad (3.20)$$

Applying the conditions on (C1)-(C5) on (3.16), we get

$$\lim_{n\to\infty}\|w_n - T_i w_n\| = 0 = \lim_{n\to\infty}\left\|S_j(Ay_n) - Ay_n\right\|, \tag{3.21}$$

For all $i = 1,2,\dots,N$ and $j = 1,2,\dots,M$.

It follows from (3.21) and the estimate on (3.7) that

$$\lim_{n\to\infty}\|z_n - Ay_n\| = 0. \tag{3.22}$$

Therefore, we conclude from the Algorithm that

$$\lim_{n\to\infty}\|t_n - y_n\| = \lim_{n\to\infty}\|A^*(z_n - Ay_n)\| = 0. \tag{3.23}$$

Also, from the Algorithm, we know that

$$\|w_n - x_n\| = \beta_n \frac{\theta_n}{\beta_n}\|x_n - x_{n-1}\| \to 0 \text{ as } n \to \infty. \tag{3.24}$$

Since $\{y_n\}$ and $\{w_n\}$ are in $H_1$ and from (3.21) we get

$$\begin{aligned}
\|y_n - w_n\|^2 &= \left\|\sum_{i=1}^{N} a_{i,n}(\overline{T}_i w_n - w_n)\right\|^2 \\
&\le \sum_{i=1}^{N} a_{i,n}\|\overline{T}_i w_n - w_n\|^2 \\
&= \sum_{i=1}^{N} a_{i,n}\left\|(\beta_{i,n} I + (1-\beta_{i,n})T_i)w_n - w_n\right\|^2 \\
&= \sum_{i=1}^{N} a_{i,n}(1-\beta_{i,n})\|T_i w_n - w_n\|^2 \to 0 \ as\ n \to \infty.
\end{aligned}$$

Therefore,

$$\lim_{n\to\infty}\|y_n - w_n\| = 0. \tag{3.25}$$

From (3.24) and (3.5), we obtain

$$\|y_n - x_n\| \le \|y_n - w_n\| + \|w_n - x_n\| \to 0 \ \ as\ n \to \infty.$$

Hence,

$$\lim_{n\to\infty}\|y_n - x_n\| = 0. \tag{3.26}$$

Also, from (3.23), (3.25) and (3.26), we get

$$\|x_n - t_n\| = \|x_n - y_n\| + \|y_n - t_n\| \to 0 \ \ as\ n \to \infty,$$

which implies that

$$\lim_{n\to\infty}\|x_n - t_n\| = 0. \quad (3.27)$$

Using the boundedness of $\{t_n\}$ and the condition on $\alpha_n$, we estimate

$$x_{n+1} = \alpha_n u + (1-\alpha_n)t_n$$
$$= \alpha_n u + t_n - \alpha_n t_n,$$

that is,

$$x_{n+1} - t_n = \alpha_n(u - t_n). \quad (3.28)$$

It follows from (3.28) that

$$\|x_{n+1} - t_n\| = \alpha_n\|u - t_n\| \to 0 \ \ as \ \ n \to \infty.$$

Thus, we obtain that

$$\lim_{n\to\infty}\|x_{n+1} - t_n\| = 0. \quad (3.29)$$

To obtain the asymptotical property of $\{x_n\}$, we combine (3.27) and (3.29) to get

$$\|x_{n+1} - x_n\| \le \|x_{n+1} - t_n\| + \|t_n - x_n\| \to 0,$$

which implies that

$$\lim_{n\to\infty}\|x_{n+1} - x_n\| = 0. \quad (3.30)$$

**Step 3:** We show that $\limsup_{n\to\infty}\vartheta_n \le 0$. Let $\{x_{n_k}\}$ be a subsequence of $\{x_n\}$ such that

$$\limsup_{n\to\infty}\langle u - x^*, x_n - x^*\rangle = \lim_{n\to\infty}\langle u - x^*, x_{n_k} - x^*\rangle.$$

Since $\{x_{n_k}\}$ is a bounded sequence, there exists a subsequence $\{x_{n_{k_j}}\}$ of $\{x_{n_k}\}$ such that $x_{n_{k_j}} \to x^{**}$. Without loss of generality, we may assume that $x_{n_k} \to x^{**}$.

**Claim:** $x^* \in \Omega$. To establish this fact, using Lemma 2.3 and the estimate in (3.21), that is, $\|w_n - T_i w_n\| \to 0$ for all $i = 1,2,\dots,N$, we obtain that $x^* \in \bigcap_{i=1}^N F(T_i)$. From (3.26) that $y_{n_k} \to x^*$. Following the linearity and boundedness of the operator $A$, we get $Ay_{n_k} \to Ax^*$. Again, using Lemma 2.3 and the fact that $\|S_j(Ay_n) - Ay_n\| \to 0$, $j = 1,2,\dots,M$, we obtain that $x^* \in A^{-1}\left(\bigcap_{j=1}^M F(S_j)\right)$. We finally have that $x^* \in \Omega = \bigcap_{i=1}^N F(T_i) \cap A^{-1}\left(\bigcap_{j=1}^M F(S_j)\right)$, establishing the claim. Hence, $x^* = P_\Omega^{H_1}u$, using (2.2), we get

$$\limsup_{n\to\infty}\langle u - x^*, x_n - x^*\rangle = \langle u - x^*, x^{**} - x^*\rangle \le 0.$$

Hence, $\limsup_{n\to\infty} \le 0$. Therefore, in the conclusion part of the Lemma 2.3, we have $\varphi_n \to 0$, that is $x_n \to x^* = P_\Omega^{H_1}$ as required.

**Case 2:** Assuming that the sequence $\{\varphi_n\}$ does not decrease at inifinity. Then, using Lemma 2.4, we define an integer sequence $\{\rho(n)\}$ for all $n \ge n_0$ (for some $n_0$ large enough) by

$$\rho(n) \coloneqq \max\{n_0 \leq k \leq n : \varphi_k < \varphi_{k+1}\}.$$

Note that $\rho(n)$ is an nondecreasing sequence such that $\rho(n) \to \infty$ as $n \to \infty$ and $\varphi_{\rho(n)} \leq \varphi_{\rho(n+1)}$ for all $n \geq n_0$. Using the estimate of (3.19) and **Step 1** we get

$$0 \leq \varphi_{\rho(n+1)} - \varphi_{\rho(n)} \leq \alpha_{\rho(n)} K,$$

where K is a constant. Since $\alpha_{\rho(n)} \to 0$, we obtain that

$$\lim_{n\to\infty}\left(\varphi_{\rho(n)+1} - \varphi_{\rho(n)}\right) = 0. \qquad (3.31)$$

Now, applying the same level of arguments in **Case 1** above, we deduce that

$$\lim_{n\to\infty}\left\|w_{\rho(n)} - T_i w_{\rho(n)}\right\| = \lim_{n\to\infty}\left\|Ay_{\rho(n)} - S_j(Ay_{\rho(n)})\right\| = 0, \qquad (3.32)$$

For all $i = 1,2,\dots,N$ and $j = 1,2,\dots,M$.

Again, using the same arguments in **Case 1**, that is, (3.22)-(3.29), we have

$$\lim_{n\to\infty}\left\|x_{\rho(n+1)} - x_{\rho(n)}\right\| = 0.$$

In the same manner, we obtain $\lim_{n\to\infty} sup\, \vartheta_{\rho(n)} \leq 0$.

Hence, using (3.19) we obtain

$$\varphi_{\rho(n)+1} \leq \left(1 - \alpha_{\rho(n)}\right)\varphi_{\rho(n)} + \alpha_{\rho(n)}\vartheta_{\rho(n)}.$$

Since $\varphi_{\rho(n)+1} > \varphi_{\rho(n)}$ and $\alpha_{\rho(n)} > 0$, we obtain

$$\varphi_{\rho(n)} \leq \vartheta_{\rho(n)}.$$

Since $\lim_{n\to\infty} sup\, \vartheta_{\rho(n)} \leq 0$, it follows that $\lim_{n\to\infty} \varphi_{\rho(n)} = 0$.
Using (3.31) , it implies that $\lim_{n\to\infty} \varphi_{\rho(n)+1} = 0$.
Since

$$0 \leq \varphi_n \leq \max\{\varphi_{\rho(n)}, \varphi_n\} \leq \varphi_{\rho(n)+1} \to 0,$$

we get $\varphi_n \to 0$, which means that $\{x_n\}$ strongly converges to $x^* = P_\Omega^{H_1} u$, as asserted. This completes the proof of the Theorem.
The following corollaries are the immediate consequences of our algorithm.

**Corollary 3.2:** Let $H_1$ and $H_2$ be two real Hilbert spaces. Let $T_i: H_1 \to H_2, 1 = 1,2,\dots,N$, and $S_j: H_2 \to H_2, j = 1,2,\dots,M$, be nonexpansive mappings respectively. Let $A: H_1 \to H_2$ be a bounded linear operator. Assume that $\Omega \coloneqq (\bigcap_{i=1}^{N} F(T_i)) \bigcap A^{-1}(\bigcap_{j=1}^{M} F(S_j)) \neq \emptyset$. We consider the problem of the following:

$$\text{find } x^* \in \Omega. \qquad (3.1)$$

For any $u, x_0 \in H_1$, We define $\{x_n\}$, the sequence generated by the following algorithms by

$$\begin{cases} w_n = x_n + \theta_n(x_n - x_{n-1}), \\ y_n = \sum_{i=1}^{N} a_{i,n}\tilde{T}_i w_n, \\ z_n = \sum_{j=1}^{M} b_{j,n} S_j(Ay_n), \\ t_n = y_n + \delta A^*(z_n - Ay_n), \\ x_{n+1} = \alpha_n u + (1-\alpha_n)t_n, n \geq 0. \end{cases} \tag{3.2}$$

where $\{a_{i,n}\}, i = 1,2,\dots,N, \{b_{j,n}\}, j = 1,2,\dots,M$ and $\{\theta_n\}$ are sequences of positive real numbers while $\tilde{T}_i \coloneqq \beta_{i,n}I + (1-\beta_{i,n})T_i$ and $\{\beta_{i,n}\} \subset (0,1), for\ i = 1,2,\dots,N.$ Then the sequence $\{x_n\}$ generated by theorem (3.2) converges strongly to point $x^* = P_{\Omega}^{H_1}(u)\ as\ n \to \infty$ if the following the conditions (C1)-(C6) are satisfied.

**Proof:** The result follows from **Theorem 3.1** above.

**4. Conclusion:** In real Hilbert space, we have established a strong convergence result under mild assumptions. We incorporated an inertial extrapolation technique which real improved the rate convergence as expected.

## References

[1] Censor, Y., Elfving, T. (1994). A multi projection algorithm using Bregman projections in a product space. Numer. Algorithm. 8(2):221–239. DOI: 10.1007/BF02142692.

[2] Byrne, C. (2002). Iterative oblique projection onto convex sets and the split feasibility problem. Inverse Probl. 18(2):441–453. DOI: 10.1088/0266-5611/18/2/310.

[3] Byrne, C. (2004). A unified treatment of some iterative algorithms in signal processing and image reconstruction. Inverse Probl. 20(1):103–120. DOI: 10.1088/0266-5611/20/1/006.

[4] Censor, Y., Elfving, T., Kopf, N., Bortfeld, T. (2005). The multiple-sets split feasibility problem and its application. Inverse Probl. 21(6):2071–2084. DOI: 10.1088/0266-5611/21/6/017.

[5] Censor, Y., Segal, A. (2009). The split common fixed point problem for directed operators. J. Convex Anal. 16:587–600.

[6] Censor, Y., Gibali, A., Reich, R. (2012). Algorithms for the split variational inequality problem. Numer. Algorithm. 59(2):301–323. DOI: 10.1007/s11075-011-9490-5.

[7] Xu, H. K. (2006). A variable Krasnosel'skii-Mann algorithm and the multiple-set split feasibility problem. Inverse Probl. 22(6):2021–2034. DOI: 10.1088/0266-5611/22/6/007.

[8] Xu, H. K. (2010). Iterative methods for the split feasibility problem in infinite dimensional Hilbert spaces. Inverse Probl. 26(10):105018. DOI: 10.1088/0266-5611/26/10/105018.

[9] Masad, E., Reich, S. (2007). A note on the multiple-set split convex feasibility problem in Hilbert space. J. Nonlinear Convex Anal. 8:367–371.

[10] Shehu, Y., Iyiola, O. S., Enyi, C. D. (2016). An iterative algorithm for solving split feasibility problems and fixed point problems in Banach spaces. Numer. Algorithm. 72(4):835–864. DOI: 10.1007/s11075-015-0069-4.

[11] Eicke, B. (1992). Iteration methods for convexly constrained ill-posed problems in

Hilbert space. Numer. Funct. Anal. Optim. 13:423–429.

[12] Cegielski, A. (2015). General method for solving the split common fixed point problem. J. Optim. Theory Appl. 165(2):385–404. DOI: 10.1007/s10957-014-0662-z.

[13] Cegielski, A., Al-Musallam, F. (2016). Strong convergence of a hybrid steepest descent method for the split common fixed point problem. Optimization 65(7): 1463–1476. DOI: 10.1080/02331934.2016.1147038.

[14] Eslamian, M., Eslamian, P. (2016). Strong convergence of a split common fixed point problem. Numer. Funct. Anal. Optim. 37(10):1248–1266. DOI: 10.1080/01630563.2016.1200076.

[15] Kraikaew, R., Saejung, S. (2014). On split common fixed point problems. J. Math. Anal. Appl. 415(2):513–524. DOI: 10.1016/j.jmaa.2014.01.068.

[16] Shehu, Y. (2015). New convergence theorems for split common fixed point problems in Hilbert spaces. J. Nonlinear Convex Anal. 16:167–181.

[17] Shehu, Y., Cholamjiak, P. (2016). Another look at the split common fixed point problem for demicontractive operators. Rev. R. Acad. Cienc. Exactas Fs. Nat. Ser. A Math. 110(1):201–218. DOI: 10.1007/s13398-015-0231-9.

[18] Tang, Y. C., Liu, L. W. (2016). Several iterative algorithms for solving the split common fixed point problem of directed operators with applications. Optimization 65(1):53–65. DOI: 10.1080/02331934.2014.984708.

[19] Padcharoen, A., Kumam, P., Cho, Y. J. (2019). Split common fixed point problems for demicontractive operators. Numer. Algorithm. 82(1):297–320. DOI: 10.1007/s11075-018-0605-0.

[20] Reich, S., Tuyen, T. M. (2019). A new algorithm for solving the split common null point problem in Hilbert spaces. Numer. Algorithm. https://doi.org/10.1007/s11075-019-00703-z

[21] Reich, S., Tuyen, T. M. (2019). Iterative methods for solving the generalized split common null point problem in Hilbert spaces. Optimization:1–26. DOI: 10.1080/02331934.2019.1655562.

[22] Reich, S., Tuyen, T. M., & Trang, N. M. (2019). *Parallel Iterative Methods for Solving the Split Common Fixed Point Problem in Hilbert Spaces. Numerical Functional Analysis and Optimization, 1–28.* doi:10.1080/01630563.2019.1681000

[23] Moudafi, A. (2000). Viscosity approximation methods for fixed-points problems. J. Math. Anal. Appl. 241(1):46–55. DOI: 10.1006/jmaa.1999.6615

[24] Halpern, B. (1967). Fixed points of nonexpanding maps. Bull. Amer. Math. Soc. 73(6):597–957. DOI: 10.1090/S0002-9904-1967-11864-0.

[25] G. 1. MINi'Y, *Monotone (nonlinear) operators* in *Hilbert space*, Duke Math. 1., 29 (1962), pp.341-346.

[26] Bauschke, H. H., Combettes, P. L. (2011). Convex Analysis and Monotone Operator Theory in Hilbert Spaces. New York: Springer.

[27] Goebel, K., Kirk, W. A. (1990). Topics in Metric Fixed Point Theory. Cambridge Stud. Adv. Math 28. Cambridge: Cambridge University Press.

[28] Mainge, P. E. (2008). Strong convergence of projected subgradient methods for nonsmooth and nonstrictly convex minimization. Set-Valued Anal. 16(7-8):899–912. DOI: 10.1007/s11228-008-0102-z.

[29] Xu, H. K. (2006). Strong convergence of an iterative method for nonexpansive and accretive operators. J. Math. Anal. Appl. 314(2):631–643. DOI: 10.1016/j.jmaa.2005.

04.082.
[30] Hicks TL, Kubicek JD (1977) On the Mann iteration process in a Hilbert space, J. Math. Anal. Appl. 59 pp. 498{504.
[31] Patrick, L: Combettes, Jean-Christophe Pesquet: Deep Neural Network Structures Solving Variational Inequalities. Optimization and Control (math.OC). arXiv:1808.07526 [math.OC] (or arXiv:1808.07526v2 [math.OC] for this version) (2019) https://doi.org/10.48550/arXiv.1808.07526.
[32] Heaton, H, Wu Fung, S, Gibali, A. et al: Feasibility-based fixed point networks. Fixed Point Theory Algorithms Sci Eng 2021, 21 (2021). https://doi.org/10.1186/s13663-021-00706-3.
[33] Combettes. PL, Pesquet,JC, "Fixed Point Strategies in Data Science," in IEEE Transactions on Signal Processing, vol. 69, pp. 3878-3905, 2021, https://doi: 10.1109/TSP.2021.3069677

[34] Jung, A: A Fixed-Point of View on Gradient Methods for Big Data. Frontiers in Applied Mathematics and Statistics, 3. doi:10.3389/fams.2017.00018.
[35] Censor Y, Gibali A, Reich S. Algorithms for the split variational inequality problem. Numer Algorithms. 2012;59:301–323.
[36] Gibali A. A new split inverse problem and application to least intensity feasible solutions. Pure Appl Funct Anal. 2017;2(2):243–258.
[37] Polyak BT (1964) Some methods of speeding up the convergence of iteration methods. Comput. Math. Math. Phys., 4, 1(17)
[38] Beck A, Teboulle M (2009) A fast iterative shrinkage-thresholding algorithm for linear inverse problems. SIAM J. Imaging Sci. 2, 183{202.
[39] Nesterov Y (1983) A method of solving a convex programming problem with convergence rate O(1/k2). Soviet Math. Doklady 27, 372{376.
[40] Qin X, Wang L, Yao JC (2020) Inertial splitting method for maximal monotone mappings. J. Nonlinear Convex Anal. in press.
[41] Luo YL, Tan B (2020) A self-adaptive inertial extragradient algorithm for solving pseudo-monotone variational inequality in Hilbert spaces. J. Nonlinear Convex Anal. in pres.